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Efficient modeling of spiral strands subjected to biaxial bending and variable axial force

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Résumé — Spiral strands exhibit nonlinear hysteretic bending behavior due to their helical geometry and interwire frictional contact interaction. In this study, an efficient framework based on computational homogenization is proposed, in which a spiral strand is replaced by a homogeneous beam with effective material properties. To obtain the macroscopic (moment curvature response) and microscopic (axial force of individual wires) responses at each integration point, rheological models are proposed, the parameters of which are easily obtained from several monotonic uniaxial bendings.
Mots clés — Spiral strand, Computational homogenization, Rheological model.

1 Introduction

Frictional interactions between steel wires within spiral strands used as mooring lines for offshore platforms induce a complex mechanical response of these strands when subjected to a bending load under tension. Due to these internal frictional interactions, the mechanical behavior of the strand at the macroscopic scale is dissipative, and the bending stiffness, which is governed by interwire slip, evolves nonlinearly with respect to curvature and tension. It is possible to simulate the effects of interwire friction using finite element simulation by considering all the individual wires that make up the rope and all the frictional contact interactions, which is called direct numerical simulation (DNS). However, this approach cannot be applied to spiral strands used as mooring lines for offshore platforms due to their size, both in terms of the number of wires and the length considered. Therefore, a mixed stress-strain computational homogenization framework is developed to identify the nonlinear constitutive model of a beam element capable of reproducing the complex hysteretic response of spiral strands subjected to bending under tension. In this framework, the spiral strand is replaced by a single beam whose material properties at each integration point are extracted from the solution of a boundary value problem (BVP) on a microscopic sample called a representative volume element (RVE). Although the computational cost of using the proposed homogenization framework would be lower compared to DNS, it is still high for large cables, as in this framework, for each integration point and at iteration of each step, a nonlinear RVE BVP should be solved. Therefore, to reduce the computational cost of the homogenization framework, rheological models are proposed to replace the solution of RVE BVP. These models are able to predict the macroscopic (moment curvature response) and microscopic (axial force of individual wires) responses of spiral strands subjected to biaxial bending and variable axial force. Using the proposed rheological models in the computational homogenization framework allows modeling of spiral strands in a few seconds which would have taken several weeks to model using DNS.

2 Direct numerical simulation (DNS)

In DNS, all wires and frictional contact interactions are modeled using finite element simulation [2]. As can be seen in Figure 1, the obtained axial-bending response of the four-layer spiral strand is in good agreement with the experimental results.

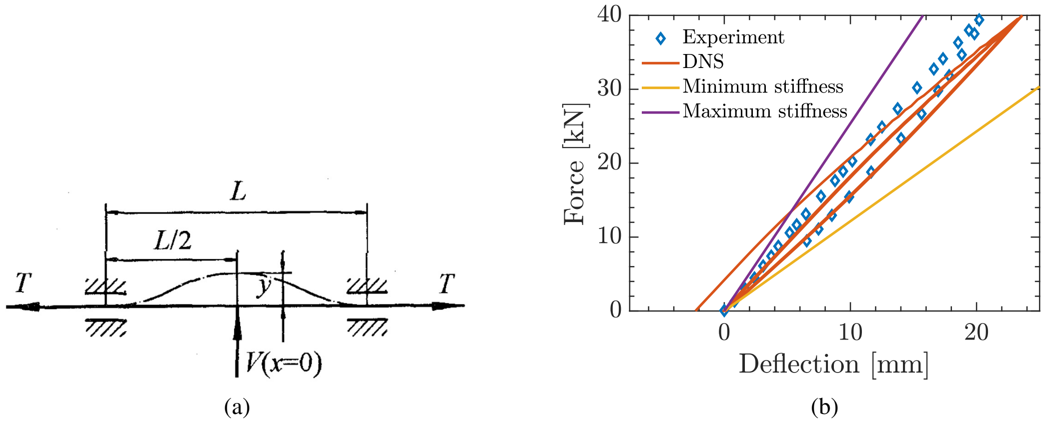


FIGURE 1 – The axial-bending experiment conducted by Papailiou [5]. a, Geometry and boundary conditions ; b, Comparison of the response of the four-layer strand obtained from DNS with experimental data.

3 A mixed stress-strain driven computational homogenization [6]

In the computational homogenization technique, a heterogeneous medium is replaced by its equivalent homogeneous medium. In this framework, the constitutive behavior for each integration point of the homogeneous medium (macro-scale model) is extracted from the representative volume element (RVE) attached to that point, where all the heterogeneities are explicitly modeled, while no explicit assumption on the macroscopic constitutive behavior is necessary. In conventional homogenization at finite strain, the macro-scale tangent modulus and first Piola-Kirschoff stress are obtained for a given macroscopic deformation gradient. In the context of cables, the spiral strand is considered as the heterogeneous medium, represented by a single beam model at the macro-scale, and a short length of the spiral strand is the RVE.

As the macro-scale model is a single beam and the mechanical properties in the longitudinal direction are dominant, the homogenization will be performed only in the longitudinal direction, and no averaging condition is considered in the transverse directions. In the proposed framework, two different beam elements are used in the macro- and micro-scales. A kinematically enriched beam element [2], which has 9 degrees of freedom and is able to capture the deformation of the cross-section, is used in the micro-scale. For the macro-scale model, the geometrically exact beam element [1], which has 6 degrees of freedom and considers a rigid cross-section, is used. In this study, cables are considered slender structures whose behavior is dominated by axial, torsional, and flexural mechanisms, and shear strains are neglected. However, it should be emphasized that shear is accounted for at the microscopic scale, as it plays a vital role in determining the macroscopic bending behavior of spiral strands. Consequently, the macroscopic strains of interest are an axial extension, a twist, and two bending curvatures. Due to the dependence of the bending stiffness of the spiral strands on the axial stress and the geometric coupling of axial force and bending curvature at finite strain, following [7], a mixed stress-strain driven homogenization framework is developed (Figure 2). In this formulation, the macroscopic strains enter the microscopic boundary value problem (BVP) as "displacement" degrees of freedom, while their work-conjugate stresses will be their corresponding dual "forces". This allows strain, stress, or mixed stress-strain driven homogenization to be performed straightforwardly. Furthermore, the macroscopic stresses and strains are naturally obtained as the solution to the microscopic BVP, without needing any averaging relation.

3.1 Axial-bending response extracted from RVE

To investigate the nonlinear RVE response due to inter-wire frictional interactions, the two-layer strand which has been studied by [4] before is subjected to an axial force equivalent to an axial strain of 10^{-3} and is then subjected to a bending curvature. The response of the RVE is presented in Figure 3. As it can be observed, the response is in very good agreement with the results from the literature.

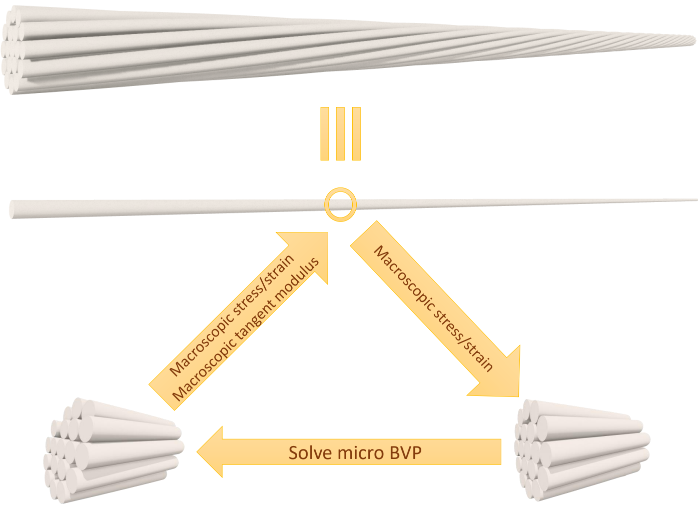


FIGURE 2 – The mixed stress-strain driven computational homogenization algorithm for spiral strands. BVP, boundary value problem.

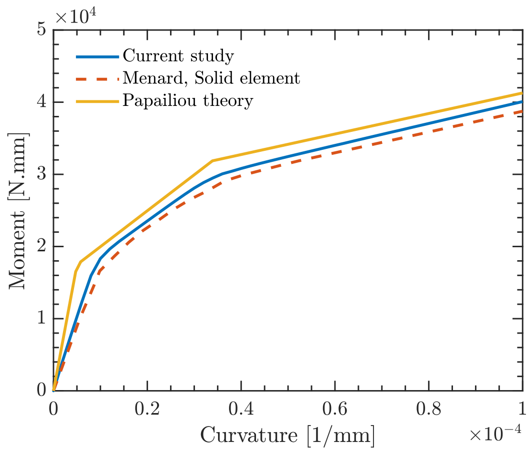
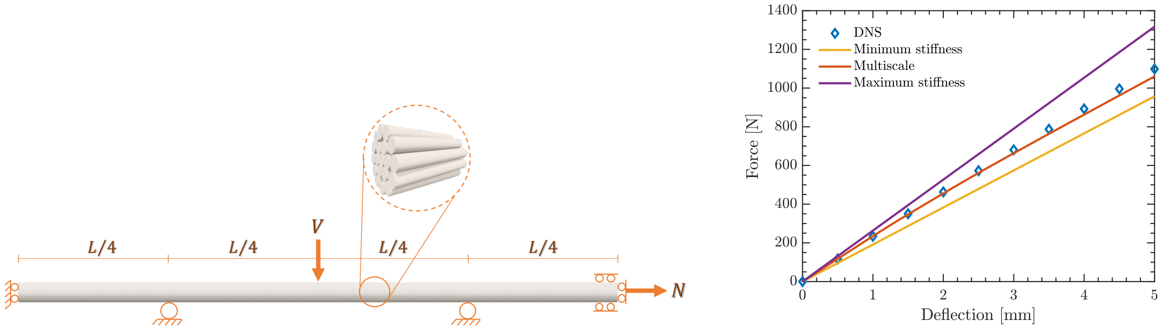


FIGURE 3 – A two-layer spiral strand behavior under combined axial force and bending curvature loading.

3.2 Verification of the homogenization scheme against DNS

In this section, the results of the multi-scale analysis are verified against direct numerical simulation (DNS). The geometry and boundary conditions of a bending experiment under constant axial force are shown in Figure 4. In order to satisfy the separation of scales, $L/4$ is considered equal to two pitches of the outermost layer.

The force vs. displacement diagram of the multi-scale and DNS models is presented in Figure 4, along with the responses considering the strand's theoretical maximum and minimum bending stiffnesses [5]. The maximum bending stiffness is computed as a function of the second moment of area of the strand, assuming its cross-section remains rigid (no sliding), while the minimum bending stiffness is taken equal to the sum of the bending stiffnesses of all constituent wires. As it can be observed, the response of the DNS is perfectly predicted by the multi-scale method. As expected, at low deflections, the response is similar to the case considering the maximum bending stiffness, and the final stiffness is equal to the minimum bending stiffness.



(a) (b)

FIGURE 4 – a, The geometry and boundary conditions of the bending under constant axial force ; b, The force-deflection responses obtained through the DNS, multi-scale analysis, and theoretical stiffnesses of the 2-layer strand.

4 Rheological models

In the framework of mixed stress-strain driven computational homogenization, a set of macroscopic stresses or strains are imposed on an RVE as boundary conditions, and their work-conjugate quantities as well as the consistent tangent modulus, which describes the relationship between increments of macroscopic strains and stresses, are obtained after solving the RVE BVP. In the context of spiral strands, the macroscopic strains are the axial strain ε, the twist K_3, and the two bending curvatures K_1 and K_2, and their corresponding macroscopic stresses are the axial stress, the torque and the two bending moments. The consistent tangent modulus, $\mathbb{C}$, is a 4×4 matrix as seen in Equation 1.

$$\mathbb{C} = \begin{bmatrix} \mathbb{C}_{\varepsilon\varepsilon} & \mathbb{C}_{\varepsilon K_3} & \mathbb{C}_{\varepsilon K_2} & \mathbb{C}_{\varepsilon K_1} \\ \mathbb{C}_{K_3\varepsilon} & \mathbb{C}_{K_3K_3} & \mathbb{C}_{K_3K_2} & \mathbb{C}_{K_3K_1} \\ \mathbb{C}_{K_2\varepsilon} & \mathbb{C}_{K_2K_3} & \mathbb{C}_{K_2K_2} & \mathbb{C}_{K_2K_1} \\ \mathbb{C}_{K_1\varepsilon} & \mathbb{C}_{K_1K_3} & \mathbb{C}_{K_1K_2} & \mathbb{C}_{K_1K_1} \end{bmatrix} \approx \begin{bmatrix} \mathbb{C}_{\varepsilon\varepsilon} & \mathbb{C}_{\varepsilon K_3} & 0 & 0 \\ \mathbb{C}_{K_3\varepsilon} & \mathbb{C}_{K_3K_3} & 0 & 0 \\ 0 & 0 & \mathbb{C}_{K_2K_2} & \mathbb{C}_{K_2K_1} \\ 0 & 0 & \mathbb{C}_{K_1K_2} & \mathbb{C}_{K_1K_1} \end{bmatrix} \quad (1)$$

The spiral strands exhibit axial-torsional and axial-bending couplings due to their helical geometry. The axial-torsional coupling has been found to be linear from numerical tests, and therefore its corresponding terms in the consistent tangent modulus, $\mathbb{C}_{\varepsilon\varepsilon}, \mathbb{C}_{\varepsilon K_3}, \mathbb{C}_{K_3\varepsilon}, \mathbb{C}_{K_3K_3}$, are constant and calculated only once. In contrast, it is well known that the axial-bending coupling is the main source of nonlinear behavior for these strands. Since this coupling is a function of the axial stress and the friction coefficient rather than axial strain and twist, most of the off-diagonal terms can be neglected, leading to the approximation given in 1, which has been verified by examining the corresponding terms of $\mathbb{C}$ obtained by the homogenization procedure on RVE. Consequently, only the terms corresponding to bending, $\mathbb{C}_{K_iK_j}$ with $i, j = 1, 2$, are assumed to be a function of the axial stress and the friction coefficient. Therefore, a rheological model capable of characterizing the biaxial bending response of a spiral strand as a function of the applied tensile force can replace the solution of the RVE BVP in the homogenization scheme if only the macroscopic response of the system is of interest. However, in many cases, the microscopic response of the strands, i.e. the axial force of individual wires, is also required. Therefore, in order to fully replace the RVE BVP solution, both the macroscopic and microscopic responses should be predicted.

4.1 Macroscopic response [3]

The rheological model capable of predicting the uniaxial bending behavior of an m-layer spiral strand subjected to a variable tensile force consists of m+1 linear springs and m frictional slider elements, as can be seen in Figure 5, and is called a unidirectional spring system. The parameters of the springs and slider elements are obtained from several monotonic uniaxial bending tests under constant tensile forces obtained from solving the RVE BVP. To extend the rheological model to describe the biaxial bending behavior of the strands, a multidirectional spring system consisting of n_θ unidirectional systems is proposed, as shown in Figure 5.

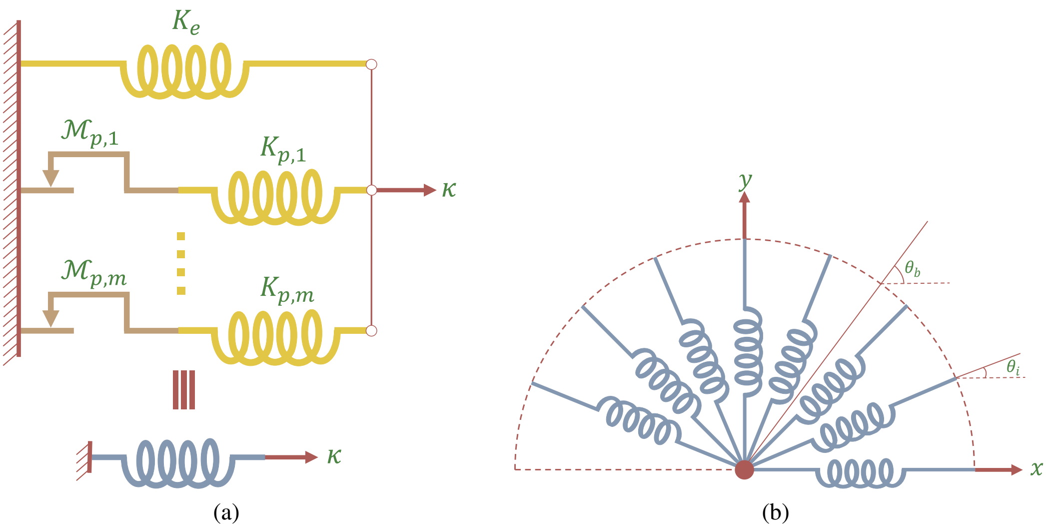


FIGURE 5 – a, The equivalent rheological models consisted of a unidirectional system to represent the uniaxial response of spiral strands ; b, The rheological model consisted of a multidirectional spring system to represent the biaxial bending behavior of spiral strands.

To verify the proposed rheological model for predicting the biaxial response of spiral strands, the two-layer strand is subjected to biaxial bending and constant tensile force, and the response of the rheological model is compared with that of homogenization. The parameters for the rheological model are extracted from the uniaxial bending responses of Figure 6. As can be seen in Figure 7, the results obtained from the rheological model are in good agreement with the homogenized responses. An important aspect of this loading is the induced anisotropy observed in the spiral strand, which has been captured by the proposed rheological model.

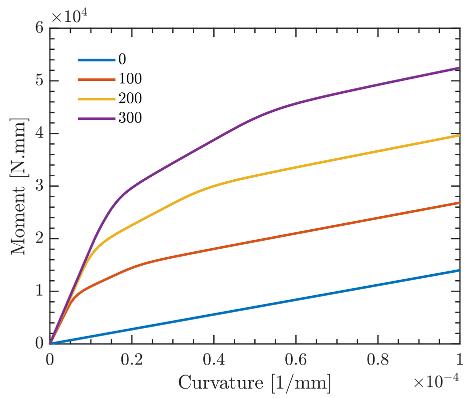


FIGURE 6 – The uniaxial bending response of the two-layer spiral strand subjected to different constant tensile stresses of 0, 100, 200 and 300 N/mm^2 used to find the parameters of the rheological model.

4.2 Microscopic response

The axial force in individual wires is required to perform fatigue life estimation for spiral strands. For this purpose, since the bending induced axial force is due to interlayer friction, the increment of the tangential force due to friction is obtained from the homogenized monotonic axial bending response of the strands as a function of the bending curvature increment. These tangential forces are then integrated to obtain the axial force of the wires. A comparison of the axial force of the wires of the two-layer spiral strand obtained from homogenization and the proposed approach when subjected to biaxial loading of Figure 7 is shown in Figure 8.

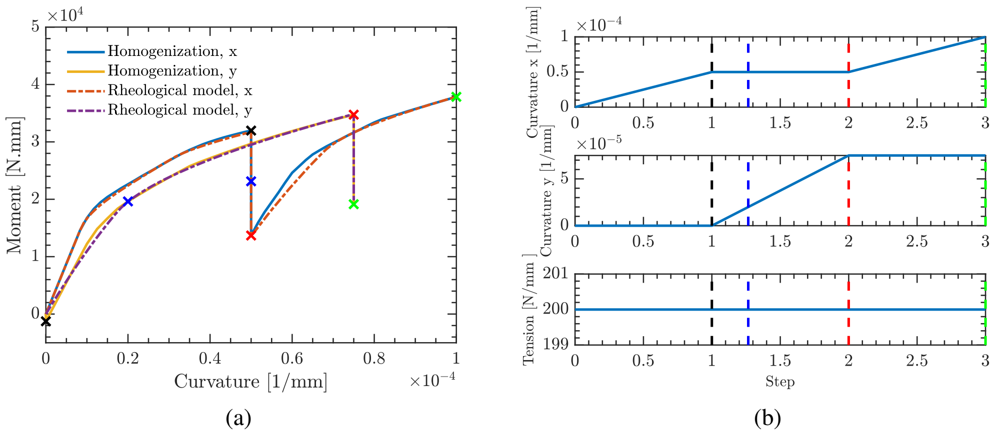


FIGURE 7 – a, A comparison of the biaxial bending response of spiral strands obtained from homogenization and from the rheological model ; b, The loading history.

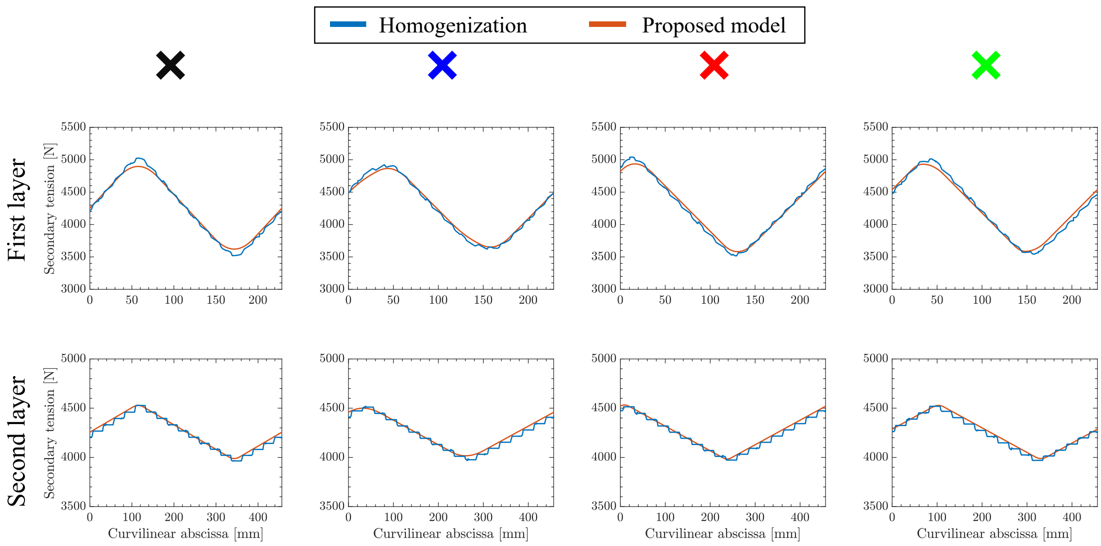


FIGURE 8 – A comparison of the variation of axial force of the wire in the first and second layer of a two-layer spiral strand under biaxial loading obtained from homogenization and the proposed approach at different loading stages. (x-axis : Curvilinear abscissa [mm] ; y-axis : Axial force [N]). Coloured crosses refer to particular loading stages identified in Figure 7.

5 Conclusion

In this study, a framework based on computational homogenization for spiral strands has been proposed to efficiently model their anisotropic nonlinear bending response. In this framework, instead of solving an RVE BVP for each macroscopic integration point, rheological models have been proposed that are capable of predicting both macroscopic and microscopic responses of these strands. Using the proposed framework, modeling of large strands would take only a few seconds, which would have taken several weeks to model using DNS.

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