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Hybrid FEM/test twin building, an electric engine case history

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Abstract — An electric engine case history illustrates issues in the hybrid use of models and test data. Having fast online models for many query evaluations of responses of interest is a traditional topic. Rather than focusing on IA related inverse methodologies, one evaluates the performance of current versions of component mode synthesis leading to direct generation of parametric reduced order models. Discussions address geometry, contact and high modal densities. Finally, model and test errors are considered for hybrid estimations of motion.
Keywords — parametric reduced model, hybrid test/FEM models, electric engine

1 Introduction

Engineering processes use models that allow predictions of the relation between inputs and outputs. Modal synthesis (or the creation of predictive reduced models based on input/output/bandwidth considerations), test/analysis correlation, updating of Finite Element Method FEM model parameters, expansion the creation of hybrid models correcting information based on test data or complementing test using model information known to be somewhat incorrect, are traditional activities of the vibration community [1].

Another classification [2] first names *virtual twins* the classical models based on the numerical implementation of mathematical equations and behavior models at the material level. *Digital twins* then correspond to the use of data, measured or simulated inputs and outputs, to build offline (meaning possibly slowly) a model that can be used online (mostly meaning that is fast). Finally *hybrid twins* combine a fast model, the possibility that its parameters may vary, an ignorance model representing what the model is assumed not to be able to predict, and an iterative (time domain) correction process known as *estimation* in the controls community.

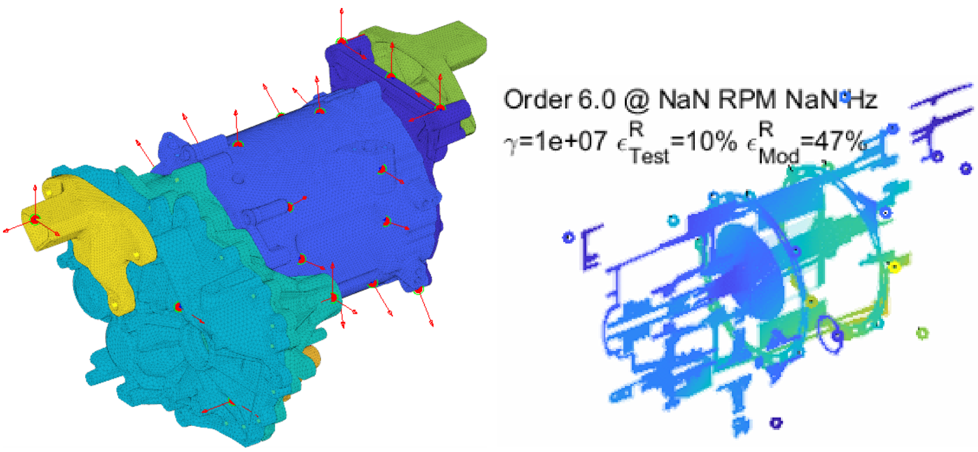

Figure 1: Left : FEM model and sensor geometry. Right : responsed estimated a 6th order harmonic

Modal synthesis methods, also referred to as parametric Reduced Order Models (ROM) or superelements, where a combination of static and eigenvalue problems form a slow learning phase ending with the production of a fast/online/reduced parametric model, share the fast aspect of a digital twin, but differ in the sense that they are direct methods that generally do not involve an optimization process. Experimental modal analysis, on the contrary, is an inverse problem that generates a fast digital twin of the input/output relation within a certain bandwidth using optimization. Updating of physical parameter in a virtual twin, uses test data to estimate physical motivated parameters and thus differs from most digital twins. Expansion methods, where a reduced parametric FEM model is combined with test data with inclusion test and FEM error models have been called hybrids for a long time [3], but do not necessarily include the time iterative aspect of estimation.

The paper will consider an electric engine, shown in figure 1, as a case study to illustrate issues thought to be important for the development of vibration models suitable for use in engineering design and validation processes.

While the physical validity of the finite element method is perfectly established, the model always has limitations that need to be analyzed. This requires describing parameters associated with limitations and generating a fast parametric model allowing practical design studies. Section 2.1 discusses parametric model reduction. Section 2.2 illustrates that geometry errors are always present and become significant for thin structures. Assemblies are nearly always idealized and associated modeling issues are discussed in Section 2.3. Finally, laminates present in electrical stacks have high modal densities which requires the use of equivalent parameters that need tuning as seen in section 2.4.

Vibration testing is typically done using point forces and sensors, illustrated by the arrows in figure 1. Using transient data is known to be poorly relevant for vibration as modes induce a correlation between spatial shapes and frequency content allowing significant reduction in the volume of data, and as measurement noise can usually be reduced by averaging the frequency domain auto- and cross-spectra. Relevant data for vibration problems thus generally is in the form of responses at given frequency known at sensors. In the illustration case, the sensors are materialized using circles and their number is clearly insufficient to get a good understanding of structural behavior. The hybrid test/FEM estimation process leading to the animated shape shown is discussed in section 3.1.

2 Fast (online) parametric models of vibration

2.1 Dynamically valid parametric models : mesh versus model size

In the case vibration problems, considered here, finite element methodologies are the reference. The resulting equations of motion take a second order form

$$[M(p)]\{\ddot{q}(t)\} + [C(p)]\{\dot{q}(t)\} + [K(p)]\{q(t)\} = [b]\{u(t)\} + [b_p]\{F_p(p, q(t), \dot{q}(t))\} \tag{1}$$

where $\{q(t)\}$ are *physical* Degree Of Freedom DOF, p the model parameters $\{u(t)\}$ the inputs signals, $[b]$ the controllability matrix relating input signals $u(t)$ to input forces at DOF and $\{F_p\}$ stresses due to parameter changes, non-linearities, ... with the virtual work induced by each of those stresses described by $[b_p]$.

To define *sensors*, and thus properly distinguish the notion of output $\{y(t)\}$ which must correspond to physical sensor shown as arrows in figure 1 from the arbitrary choice of $\{q\}$ DOF, one then builds the FEM observation equation of the form

$$\{y(t)\} = [c]\{q(t)\} \tag{2}$$

with (1) known as the evolution equation in system modeling theory and the observation equation (2) rarely explicited in FEM environments. The linear observation formulation is adapted for all the typical modal analysis sensors : accelerometers, laser vibrometer, strain gauges and load cells, piezoelectric patches, ... The building of the observation matrix, requires placement of sensors in the FEM mesh which is called the topology correlation phase of experimental modal analysis [4].

Classical component mode synthesis [5] has firmly established that for a nominal set of parameters p_0 choosing a frequency band of interest and choosing the displacement or force input localization (defining b) is sufficient to build an accurate reduced order model without need for further error control. This approach has been extended to parametric loads [6] considering non constant matrices $[M(p)], [K(p)]$ and general parametric forces spatially described by $[b_p]$. It should be noted that parametric forces include the non-constant matrix case and in the case of test/analysis correlation forces at sensor locations needed for a good use of model based expansion (see MDRE [7, 8] or CRE [9]).

Parametric ROM thus obtained have the advantage of being physically motivated and allow fast evaluations. In the case of interest, the full electric machine is considered, including over 1,700,000 elements for around 9 million DOF. This model is rather heavy considering the fact that for a 10kHz frequency band, around 1,300 modes are needed. The full real mode basis occupies 87 GB. Model reduction is thus mandatory to obtain reasonable computation times. Computations of the reduction phase are realized in ABAQUS with the AMS option, that implements its automated multilevel substructuring resolution procedure. Output sampling is also used in combination with a dedicated display strategy to optimize output storage. Integration of parameter variation requires basis enhancement, that can be implemented with residual modes associated to parametric loads which are concentrated in most implementations but can be distributed [6].

Once a parametric superelement generated, the optimal strategy for evaluating the frequency response is notably dependent on number of DOF (reduction basis size), number of frequencies of interest, number of inputs and outputs. Taking a representative case with 1723 DOF, 5 inputs, 36 outputs. Direct computations take 42ms/freq or 84s for 2000 points. Using paged versions of functions uses better vectorization and parallelisation but only provides a speedup of 1.5 while requiring notably more memory. The MATLAB `freqresp` takes a slower 144s mostly because it uses a first order form and thus 3446 states. Using the intermediate step of a diagonalization on the basis of complex modes, introduces an initialization phase of 16s, but then allows a computation of all frequency points in about 10ms. Inversion matrices at different frequencies is thus only relevant below 400 evaluated frequencies. Another classical strategy for further speedup is to use multiple levels of CMS [10, 11].

While computation time was the first historical motivation, the second use of interest of reduced models is to allow on the fly restitution of all physical quantities (displacements, strains, loads, ...). The real mode basis at all DOF uses 87 GB, but the display of this basis on the sections shown in figure 1 right only uses 65 MB, and storing the frequency response in reduced coordinates at 5000 frequency points and 100 design points only requires 12 GB, thus allowing a real time navigation within the parametric domain following user mouse movements which is actually a necessity to gain an understanding of complex situations such as that illutrated in figure 4.

2.2 Geometry errors

The first classical modeling activity is to generate a FEM model giving a good approximation of the true geometry. It has been shown that getting the geometry right is the first issue [4]. As an illustration, figure 2 shows a typical thin carter component where the notion of true geometry is evaluated by comparing the differences between a very refined mesh and one using the mesh size choice base on wavelength at target frequency. The errors shown in figure 2 are not exceeding 0.5 mm and may thus seem small, but they induce a frequency shift of between 2 and 3%.

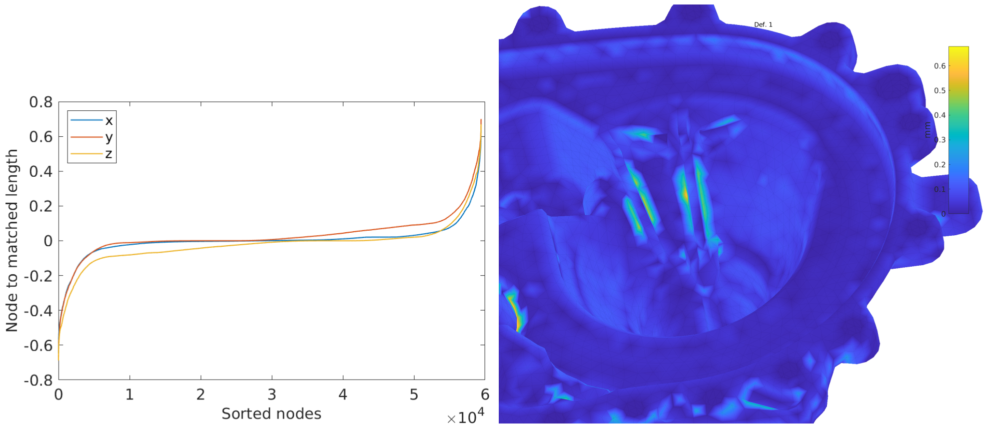

Figure 2: Geometry errors on the details of the connector housing. Left: directional error sorted by magnitude. Right: error under the output shaft area

Such errors are not easily accounted for, even though geometry morphing approaches [12] are compatible with model reduction.

2.3 Contacts between assembled components

A second recurring source of model errors is the representation of contacts. Figure 3 illustrates the case of two parts assembled through bolts. The bolts are known to generate normal stresses that decrease with distance to the bolt [13], as a result surfaces away from the bolt, shown in blue in Figure 3 left, are typically lightly or not loaded. Usual commercial codes do not allow for a refined surface coupling modeling, the flanges having to be either glued or (fully bonded) or free – a preloading step can refine the actual coupling surface area, but not the tangent formulation itself. Furthermore Craig-Bampton, often being the only considered reduction, one has to limit interaction points to its minimum to avoid excessive increase of interface sizes (as interface reduction is still not widely deployed outside AMLS based eigensolvers) and the habit is to consider the know contact areas (shown in red here) as rigid, which can be far from usual situation in particular for shear.

Frequencies are sensitive the relative variations of both the normal contact K_n and tangential adhesion K_t surface stiffness densities. Figure 3 right thus illustrates RG1 housing frequency evolution as function of K_t for multiple values of K_n. Experimental frequencies are materialized with horizontal dashed lines and the optimization of parameters will be discussed.

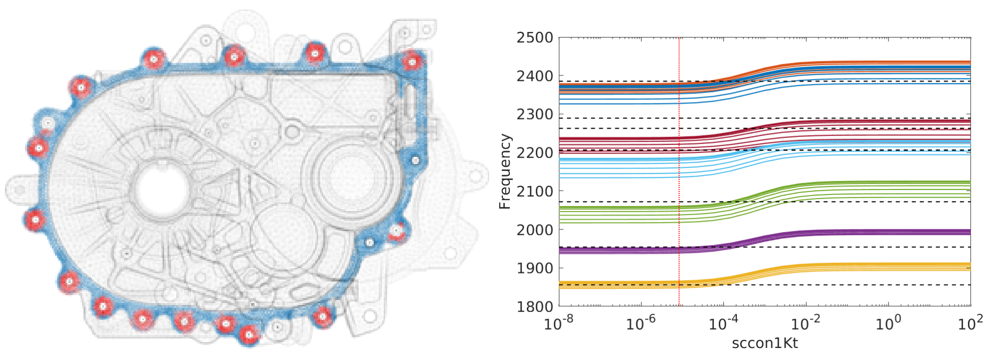

Figure 3: Assembled reductor housing contact surface localization. Left: refinement separating areas under bolt heads from surfaces that are not directly loaded, Right: influence of K_t for multiple values of K_n

The gearbox is also a strong source of uncertainty in contact properties illustrated in Figure 4. The shapes of different operating ranges differ notably and the discussion will illustrate how navigating the evolution of frequencies with parametric changes provides useful insight.

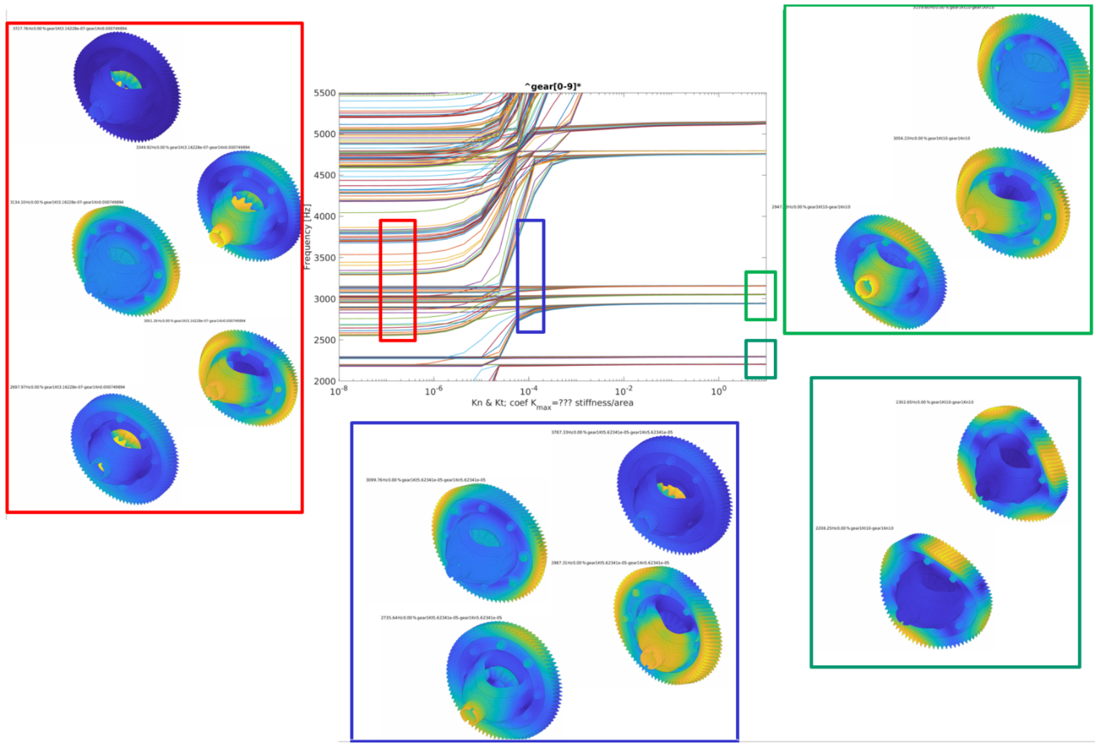

Figure 4: Frequency localization as function of loose gear back support contact

2.4 High modal density in laminated stators

An electric engine typically contains a stator composed a laminated stack supporting the electrical winding and encased in a carter. Detailed modeling would require to mesh individual laminates and their contacts, as well as the wires which are typically well supported within the carter but have relatively contact free, and thus flexible, parts outside of the stack. It is thus necessary to build equivalent models for the stack [14] and adjusting their properties is an issue. The sample mode shown in figure 5 left illustrates the local bending modes and the potential influence of contact. The right plot illustrate the strong sensitivity of many modes and the presence of additional experimental modes visualized as dotted lines.

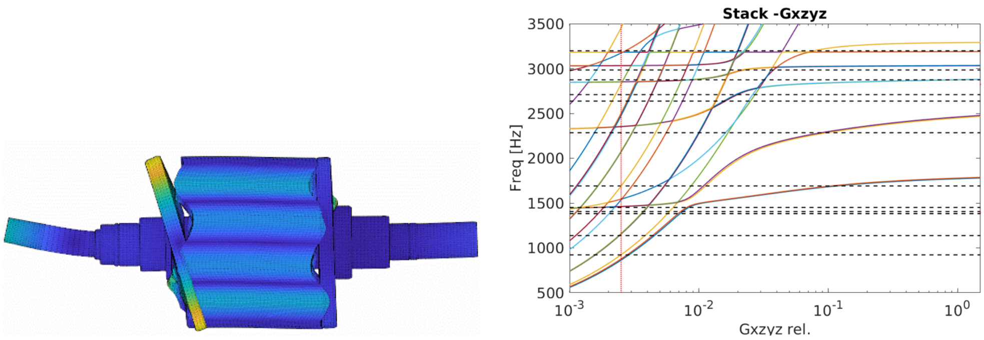

Figure 5: Stator stack modal properties. Left : sample mode. Right: frequency variations associated to the homogenized orthotropic constitutive law.

3 Combining test data and parametric FEM models

3.1 State and parameter estimation using a multi-objective optimization

Once a parametric model showing realistic parameter variations obtained, it is desirable to update or determine a choice of parameters that minimizes a distance between test and analysis. A general class of methods, see among many [15, 16, 17], formulates expansion as a multi-objective minimization problem [18, 19] combining modeling and test errors and using the frequency information as in dynamic expansion. For a measured vector $\{y\}$, a state estimate/expanded vector $\{q_{Exp}\}$, one defines an objective function

$$J(\{y\}, \{q_{exp}\}, p, \gamma) = \|R(q_{exp}, p)\|_K^2 + \gamma \quad \|\{y_{Test}\} - [c]\{q_{exp}\}\|_Q^2 \tag{3}$$

where

- $R(q_{exp}, p)$ is a *modeling error* residual, which depends on model parameters p and the expanded shape. Natural dynamic residuals are $\{R\} = [Z(\omega, p)]\{q_{exp}\}$ for modeshapes and $\{R\} = [Z(\omega, p)]\{q_{exp}\} - [b]\{u(\omega)\}$ for frequency response to the harmonic load F.
- $\|\|_K$ designates an energy norm. The motivation of this norm is explicit in the name Minimum Dynamic Residual Expansion [16] but is also motivated differently in the *Error in Constitutive Relation* work [15]
- $\{y_{Test}\} - [c]\{q_{exp}\}$ is the usual test residual measuring difference between measurement and observation of the expanded shape
- $\|\|_Q$ designates a measurement error norm. Early work on the choice of Q assumed nothing and thus used an Euclidian norm. Assuming Gaussian measurement noise and thus relating Q to the noise variance is the usual approach in Kalman filtering. Illustrations will however be given on the fact that bias is often larger than variance so that other error characterizations may be used [17]. The use of *energy* metrics is mentioned in many papers but does not represent the fact that measurement errors are not related to a form of energy.
- γ corresponds to the relative weight between the two objectives. The relative weighting was identified as an issue in very early work [20] and has been the object of much attention since. The illustrations in section **??** will show that our experience requires a log-scale search for the optimal value.

The outputs of the optimization are the expanded shapes q_{exp}, the choice of the multi-objective weighting γ and possibly the set of model parameters p (updated model), that minimizes the objective.

Illustrations will be similar to the expanded shape shown in figure 1 right and address localization of measurement problems, computational performance, and parameter updating.

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