CSMA 2024
16ème Colloque National en Calcul des Structures
13-17 Mai 2024, Presqu'île de Giens (Var)

On the benefits of a multiscale strategy to model-order reduction for frictional contact problems

D. Zeka[1], P.-A. Guidault[1], D. Néron[1], M. Guiton[2]

[1] *Université Paris-Saclay, CentraleSupélec, ENS Paris-Saclay, CNRS, LMPS - Laboratoire de Mécanique Paris-Saclay, 91190, Gif-sur-Yvette, France. {donald.zeka, pierre-alain.guidault, david.neron}@ens-paris-saclay.fr*
[2] *IFP Energies nouvelles, Rond-Point de l'échangeur de Solaize - BP 3, 69360 Solaize, France, martin.guiton@ifpen.fr*

Résumé — In this work, the efficiency of a multiscale strategy based on a domain decomposition method (DDM) for model-order reduction of friction contact problems is presented. The resolution strategy is based on the LATIN method combined with the Proper Generalized Decomposition (PGD). The effectiveness and possible improvement points of the method will be illustrated on a one-dimensional frictional benchmark problem which features a large frictional contact interface and sliding front.
Mots clés — multiscale strategy, model-order reduction, frictional contact.

1 Introduction

The direct numerical resolution of large-scale, time-dependent non-linear problems still nowadays remains a cumbersome challenge, especially for frictional contact problems. There are in this regard a range of applications where one should not negotiate on the quality and accuracy of the solution. It is the case, for example, of helically wounded steel wires, widely encountered in the energy and power transmission fields. The life prediction of such structures involving fretting fatigue phenomena between their constituent wires is crucial. However, wire-scale simulations for varying tension and bending during service life require effective computational and modeling strategies [1]. Examples include multigrid acceleration strategies or DDM techniques for numerical resolution parallelization [3]. A different path consists in adopting reduced-order models (ROM) methods to decrease the computational cost by seeking the solution of the given problem in a reduced-order basis (ROB). Nevertheless, a ROB may not easily and efficiently capture non-regular and propagating multiscale phenomena that occur at contact interfaces : sliding, sticking and separation zones being difficult to represent. Indeed, for specific problems where high accuracy of local contact quantities is required, ROM alone may not guarantee high accuracy while ensuring at the same time a significant decrease in computational cost. As highlighted in [4, 5], frictional contact problems present a multiscale content, with global modes on the structural level and localized modes bringing corrections at the contact interfaces. This suggests that proposing a model reduction method within a multiscale approach may be truly beneficial for problems of this type.

Here, the mixed DDM strategy based on the LATIN method [8] is considered. The LATIN (LArge Time INcrement) is a non-incremental solver for non-linear problems which iterates on the whole space-time domain with an iterative scheme which shares similarities with augmented Lagrangian formulations and Uzawa-like algorithms, known to be robust for contact problems [13]. The mixed nature of the method, with both interface displacements and interface forces as unknowns, enables one to deal with interfaces characterized by different behaviours with a single resolution method. Moreover, the global space-time approach makes the method suitable for on-the-fly model-order reduction based on the PGD [9]. A multiscale version of the LATIN-based mixed DDM was introduced in [10, 7, 6]. However, no particular focus has been paid on frictional problems per se, with a few contact interfaces surrounded by other material non-linearities that reduce the influence of the frictional interfaces on the whole structure.

This work is intended to highlight how and to what extent a multiscale approach, in particular one based on DDM, may be helpful in efficiently solving frictional contact problems with a model reduction approach. To this end, the multiscale strategy is briefly presented and applied to a one-dimensional frictional contact problem. The strengths of the method are presented and possible strategies for improvement are proposed.

2 A multiscale strategy for frictional contact problems

The reference problem of the equilibrium of an elastic structure, under the assumptions of small displacements and an isothermal quasi-static state, occupying the space domain Ω on the time interval $[0,T]$, is considered. The structure is subjected to time-dependent body forces $\underline{f}_d$, imposed loads $\underline{F}_d$ on a portion $\partial_2\Omega$ of the boundary as well as prescribed displacements $\underline{u}_d$ on the complementary part $\partial_1\Omega$. The basic idea consists in describing the structure as an assembly of *substructures* and *interfaces*, where each substructure has its own variables and equations.

A substructure $\Omega_E \subset \Omega$, $E \in \mathbf{E}$, with $\mathbf{E}$ being the set of substructures, is subjected to the action of its environment (i.e., the neighbouring substructures $\mathbf{V}_E$) by a force distribution $\underline{F}_E$ and a displacement field $\underline{W}_E$ on its boundary $\partial\Omega_E$. The kinematic admissibility condition for $\underline{W}_E$ is denoted with $\mathbf{E}_{E,\text{ad}}$ and $\mathbf{F}_{E,\text{ad}}$ represents the static admissibility conditions for $\underline{F}_E$. The admissibility of the couple $\mathbf{s}_E = (\underline{W}_E, \underline{F}_E)$ is denoted by $\mathbf{S}_{E,\text{ad}}$ and includes the verification of the linear elastic constitutive relation $\boldsymbol{\sigma}_E = \mathbf{K} : \boldsymbol{\varepsilon}_E$, with $\mathbf{K}$ being Hooke's tensor. An interface $\Gamma_{EE'}$ between substructures Ω_E and $\Omega_{E'}$, on the other hand, must verify equilibrium conditions and the interface constitutive behaviour $\boldsymbol{b}_{EE'} = \underline{0}$, which depends on the nature of the interface (perfect interface, contact interface, boundary condition interface etc.).

The introduction of a multiscale strategy is operated at the interface level. The interface unknowns $\mathbf{s}_E = (\underline{W}_E, \underline{F}_E)$ are split into $\mathbf{s}_E = \mathbf{s}_E^M + \mathbf{s}_E^m$ prior to any discretization, where $\mathbf{s}_E^M$ is the set of the *macro* quantities and $\mathbf{s}_E^m$ the complementary set of *micro* quantities. The macroscale is defined by the characteristic length of the interfaces, which is a priori greater than the discretization on the microscale. The most important feature of the multiscale strategy is that the equilibrium conditions at the interfaces are partially verified a priori on interface macroquantities :

$$\forall E \in \mathbf{E}, \forall E' \in \mathbf{V}_E,\ \underline{F}_E^M + \underline{F}_{E'}^M = \underline{0}. \tag{1}$$

The corresponding admissibility space is denoted with $\mathscr{F}_{\text{ad}}^M$. For further details on the multiscale strategy one may refer to [7, 6].

2.1 The multiscale strategy within the LATIN framework

The decomposition into substructures and interfaces with their own equations to be satisfied leads to partition the reference problem into two manifolds of solutions. The manifold $\mathcal{A}_d$, corresponding to the set of the linear and possibly global equations pertaining to the substructures, and the manifold Γ representing the set of the local and possibly non-linear equations at the interface level. With this partitioning, it is possible to apply the LATIN method [8]. The solution is therefore sought iteratively with a two-search direction alternate algorithm between $\mathcal{A}_d$ and Γ, as shown schematically in Fig. 1.

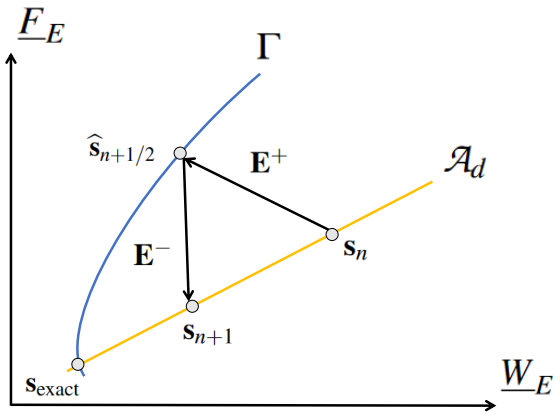


FIGURE 1 – LATIN iterative scheme.

Knowing the solution $\mathbf{s}_n \in \mathcal{A}_d$ at iteration n, the first step consists in finding $\widehat{\mathbf{s}}_{n+1/2} \in \Gamma$ by following the ascent search direction $\mathbf{E}^+$, which represents the *local stage* of the LATIN. The local stage problem (2.1) is solved at the interface level taking into account the different nature of each interface. For frictional

contact interfaces, Signorini non-penetration conditions and Coulomb's frictional law have to be verified. Other interface behaviours such as perfect linkage interfaces or boundary conditions interfaces can be also taken into account thanks to the mixed nature of the strategy.

Problem 2.1 (*local stage*)**.** Find $\widehat{\mathbf{s}} = \{\widehat{\mathbf{s}}_E\}_{E\in\mathbf{E}}$, verifying :

$$\Gamma \left\| \begin{array}{l} \text{equilibrium of forces : } \widehat{\underline{F}}_E + \widehat{\underline{F}}_{E'} = \underline{0}, \\ \text{interface constitutive behaviour } \boldsymbol{b}_{EE'}(\widehat{\underline{W}}_E, \widehat{\underline{W}}_{E'}, \widehat{\underline{F}}_E, \widehat{\underline{F}}_{E'}) = \underline{0}, \\ \text{search direction } \mathbf{E}^+ : \widehat{\underline{F}}_E - \underline{F}_E - k(\widehat{\underline{W}}_E - \underline{W}_E) = \underline{0}. \end{array} \right.$$

From $\widehat{\mathbf{s}}_{n+1/2} \in \Gamma$, by following a descent search direction $\mathbf{E}^-$, the solution $\mathbf{s}_{n+1} \in \mathcal{A}_d$ is found in the *linear stage* by solving a linear elastic problem (2.2) at the substructural level.

Problem 2.2 (*linear stage*)**.** Find $\mathbf{s} = \{\mathbf{s}_E\}_{E\in\mathbf{E}}$, verifying :

$$\mathcal{A}_d \left\| \begin{array}{l} \text{the admissibility of } \mathbf{s}_E : \mathbf{s}_E \in \mathbf{S}_{E,\text{ad}}, \\ \text{the admissibility of } \underline{F}^M : \underline{F}^M \in \mathscr{F}^M_{\text{ad}}, \\ \text{search direction } \mathbf{E}^- : \underline{F}_E - \widehat{\underline{F}}_E + k(\underline{W}_E - \widehat{\underline{W}}_E - \widetilde{\underline{W}}^M_E) = \underline{0}. \end{array} \right.$$

In the search direction $\mathbf{E}^-$, the quantity $\widetilde{\underline{W}}^M_E$ represents a Lagrange multiplier introduced to enforce the equilibrium conditions (1). It is computed at every iteration by solving a *coarse-scale problem* on the whole structure. The coarse-scale problem allows macro quantities to be propagated among all substructures and ensures numerical scalability over the number of substructures. [6]. The quantity k appearing in the search direction equations is the *search direction parameter*, homogeneous to a stiffness. The value of k affects the convergence rate of the problem, but not the solution at convergence. Reference values for k usually adopted in each substructure are given by the stiffness of the substructure itself.

In order to control the convergence of the LATIN algorithm, one can build error indicators based on the distance between two consecutive solutions belonging to each of the two manifolds. The usual error indicator adopted, introduced in [8], is the LATIN indicator :

$$\eta := \frac{\sum_E \|\mathbf{s} - \widehat{\mathbf{s}}\|^2}{\frac{1}{2}\sum_E (\|\mathbf{s}\|^2 + \|\widehat{\mathbf{s}}\|^2)}, \text{ with } \|\square\|^2 := \int_{\partial\Omega_E \times [0,T]} (k\underline{W}^2 + \frac{1}{k}\underline{F}^2) \mathrm{d}\Gamma \mathrm{d}t. \tag{2}$$

Such an error indicator characterizes the global distance in space and time between two consecutive solutions of the linear and local stages blending together displacements and interface forces with the search direction parameter k. The LATIN indicator, by definition, does not allow to accurately control the solution locally in space and time. A more stringent indicator has been proposed in [12]. In both cases, however, the indicator depends on k. In [11], a novel error indicator was introduced, which is based solely on the non-satisfaction of the constitutive behaviour at the interfaces.

2.2 Approximation of the linear stage : Proper Generalized Decomposition

The linear stage at iteration $n+1$ consists in solving a set of linear problems for each substructure over all the space-time domain. A PGD approximation of the linear stage can be introduced in order to speed up the computations [6]. First of all, the solution $\mathbf{s}_{n+1}$ at the current iteration can be equivalently written in term of corrections with respect to the previous iteration solution $\mathbf{s}_{n+1} = \mathbf{s}_n + \Delta\mathbf{s}$, with $\Delta\mathbf{s} = (\Delta\underline{W}, \Delta\underline{F})$. Given the linearity of the equations pertaining to the manifold $\mathcal{A}_d$, for each substructure Ω_E the admissibility conditions can be equivalently written in terms of corrections, as well as the search direction $\mathbf{E}^- : \Delta\underline{F}_E + k\Delta\underline{W}_E - \widehat{\underline{\delta}}_E = 0$. The quantity $\widehat{\underline{\delta}}_E$ is a residual in the search direction, it is a quantity known at this stage that depends on the iterates coming form the previous local and global stages and on the Lagrange multiplier $\widetilde{\underline{W}}^M_E$.

Introducing PGD consists in looking for a space-time separated representation in space and time for the corrections of forces and displacements, that is $\Delta\underline{W}_E = \underline{Z}(\underline{x})\varphi(t)$ and $\Delta\underline{F}_E = \underline{G}(\underline{x})\varphi(t)$. If this separated representation is required to verify admissibility conditions, the following constraint has to be verified for space modes :

$$\int_{\Omega_E} \varepsilon_E(\underline{Z}) : \mathbf{K} : \varepsilon_E(\underline{Z}^*) \mathrm{d}\Omega = \int_{\partial\Omega_E} \underline{G} \cdot \underline{Z}^* \mathrm{d}\Gamma \quad \forall \underline{Z}^* \in \mathbf{E}_{E,\text{ad}}, \tag{3}$$

By requiring a separated representation of the unknowns, as well as the admissibility condition (3) for space modes, the search direction equation can only be verified in a weak sense through the following minimal residual problem [6, 11] :

Problem 2.3. Find $(\underline{Z},\underline{G}) \in \mathbf{E}_{E,\mathrm{ad}} \times \mathbf{F}_{E,\mathrm{ad}}$ and $\varphi \in \mathcal{L}^2_{[0,T]}$ satisfying

$$(\underline{Z},\underline{G},\varphi) = \arg \min_{(\overline{G},\overline{Z},\overline{\varphi})} \|(\overline{G}+k\overline{Z})\overline{\varphi} - \widehat{\underline{\delta}}_E\|^2_{\partial\Omega_E\times[0,T]}, \tag{4}$$

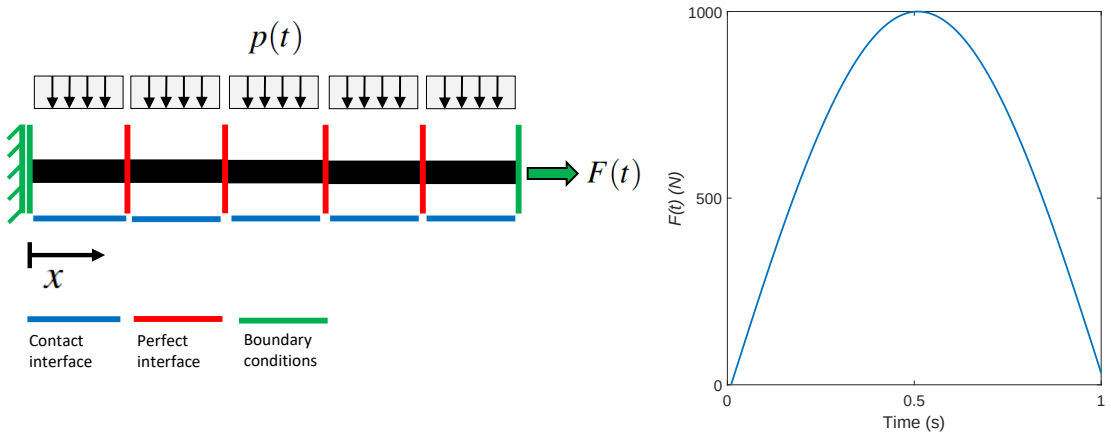
as well as the admissibility condition (3).

The resolution of (4) is usually performed with a fixed point iterative strategy. The first step consists in computing the space modes knowing the time mode φ from the previous step. Then φ is updated knowing the space modes from the first step. The process is stopped after few iterations.

Such a progressive PGD decomposition where new modes are added along the iterations, is generally not optimal, which means that a large number of modes may be required for a given accuracy. To improve the quality of the basis and control its size so that it does not grow uncontrollably, time modes updating [9] and downsizing algorithms [4, 14] can be adopted and will be briefly explained in the next section.

3 Numerical application

As a simple but very demonstrative example, an application on a one-dimensional frictional contact problem is reported. The benchmark problem consists of a clamped bar subjected to a time dependent traction loading $F(t)$ on the free side (Fig. 2b). The bar is in contact with a frictional interface by means of a normal pressure $p(t)$ acting on it (Fig. 2a). In practice, it is assumed that the bar is always in contact with the surface due to the pressure p.



(a) (b)

FIGURE 2 – Benchmark problem set : sketch of the problem (a) and time evolution of the load $F(t)$ (b).

A monoscale monostructure model reduction analysis of this test case was performed in [14]. It was shown that the problem presents a low reducibility, especially for frictional forces and, because of this, a lot of PGD modes on the whole contact interface need to be computed. This may lead to loss of efficiency and poor local accuracy for contact quantities.

Here, the benefits that a domain decomposition approach can bring, and especially the computational gain in introducing a multiscale aspect, are studied. The structure is therefore decomposed into several substructures that interact with each other through connecting perfect interfaces and subject also to the frictional contact interface, as in Fig. 2a in the case of decomposition into 5 substructures.
The used parameters are shown in Tab. 1. A reference search direction $k = ES/l_{sub}$ was adopted for both frictional and perfect interfaces, with l_{sub} being the length of a substructure.

Parameters	
Young modulus, E	210 GPa
Bar cross section, S	3.14 mm^2
Bar length, L	1 m
Number of DOFs, N_x	51
Number of time steps, N_t	101
Time interval, T	1 s
Friction coefficient, μ	0.3
Pressure load, $p(t)$	5000 N/L

TABLE 1 – Used parameters for the benchmark problem.

3.1 Performance of the multiscale approach

Fig. 3 shows, in the case of 10 substructures, the behavior of LATIN indicator (2) in the monoscale version and in the multiscale case. The gain in convergence with the introduction of the multiscale aspect is remarkable. The multiscale approach, in particular, allows for a huge convergence gain in the first iterations, where macroquantities converge rapidly. Subsequently, the quantities converge at the local microlevel. In this second stage the monoscale and multiscale approach exhibit the same convergence rate. To highlight the effect of the multiscale approach on the solution of the problem, Fig. 4 shows, in the case of 10 substructures, the solution of the problem after 10 iterations of the LATIN method for the monoscale approach and the multiscale one, compared to the reference solution at time $t = 0.5s$ and $t = 1s$. It is evident how, after a a few iterations, the multiscale approach succeeds in capturing the global behavior of the problem very well. Subsequently, further iterations are needed for the microquantities to converge, especially at the sticking-sliding discontinuity zones. The monoscale approach, on the other hand, turns out to be far from the reference solution, with the loading boundary condition still not fully propagated along all the substructures.

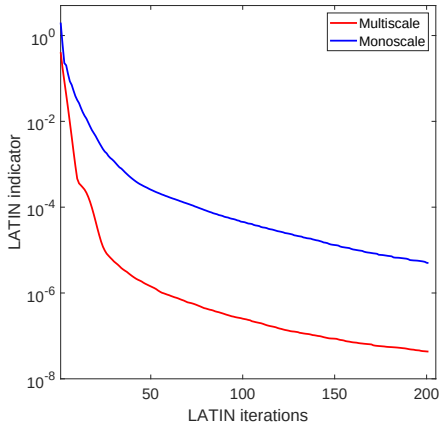
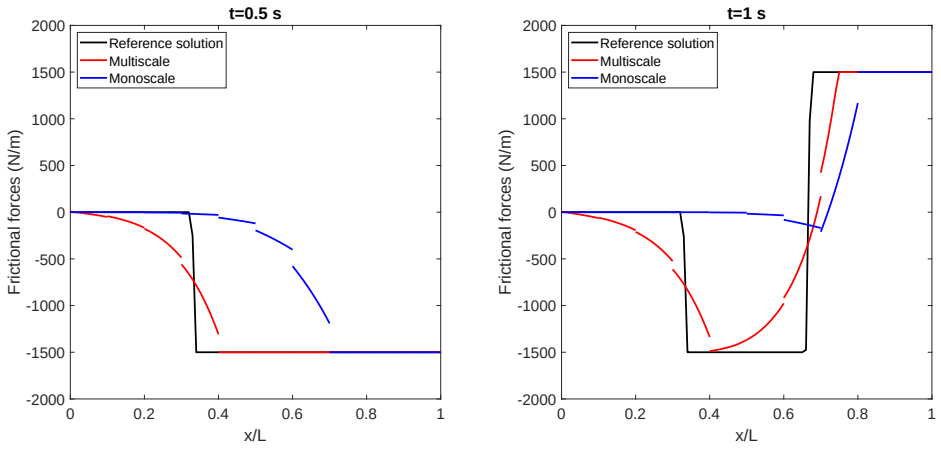


FIGURE 3 – LATIN indicator evalution for the monoscale and the multiscale approach.

These results show indeed how the multiscale approach allows for significant gain, especially in the first iterations. However, local convergence of microquantities still continues to maintain the same convergence rate as the monoscale approach, which may still lead iterate a considerable number of times in order to have an accurate computation of local quantities. One method to improve this weakness may be to update the search direction parameter k, taken constant in space and time, according to the contact conditions once the contact status has come to convergence. The search direction parameter would then result as a function of space and time $k(\underline{x},t)$. Updating the search direction can be, however, computationally burdensome since it would require to recompute the linear operators of the LATIN. Despite this, still updating few time after the contact status has come to convergence may be beneficial, especially if one can provide a separated representation of $k(\underline{x},t)$ to facilitate the integration of the operators.



(a) (b)

FIGURE 4 – Frictional contact forces distribution after 10 iterations of the LATIN for the monoscale and the multiscale approach at time $t = 0.5s$ (a) and $t = 1s$ (b).

3.2 Preliminary SVD analysis and introduction of PGD

Before proceeding with the application of PGD in the test case, an a posteriori SVD analysis of the previous problem in the case of 5 substructures is given. Fig. 5 shows the trend of the singular values for the frictional contact forces in the different substructures, numbered from 1, which corresponds to the clamped substructure, to 5, corresponding to the substructure where the external force is applied. Different substructures subjected to different sticking-sliding conditions show different potential reducibility. The first two, more under sticking conditions, exhibit singular values that decrease very rapidly. The last three, on the other hand, where a large sliding front is present, display a very slow trend in singular values decreasing

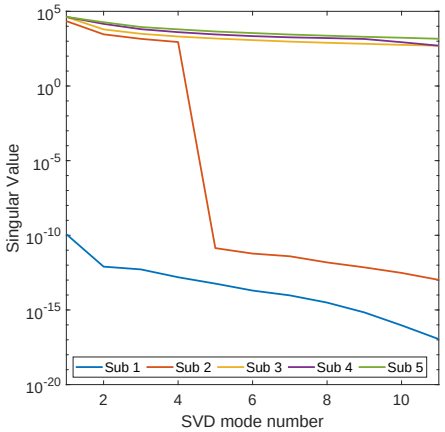


FIGURE 5 – LATIN indicator evalution for the monoscale and the multiscale approach.

In Fig. 6 is shown a comparison between space modes of frictional forces for substructures 4 and 5. A more global aspect of the first modes, more energetic and, one might say, at “low-frequency" is evident, compared with subsequent modes that bring more local, “high-frequency" corrections. The contribution of the first modes in the multiscale approach is well captured in fact by the macroproblem for the whole structure. This corresponds in fact to the rapid initial convergence rate at the beginning in Fig. 3 and can be visualized as well in the solution plot of Fig. 4 where some information is still missing locally in the sticking-sliding transition zones. Moreover Fig. 6 highlights another interesting aspect. Indeed, substructures 4 and 5 exhibit similar sticking-sliding conditions and evolution over time, for this reason their modes present similar shapes. This suggests the idea of being able to share the PGD basis between similar subdomains along the LATIN iterations [6, 11].

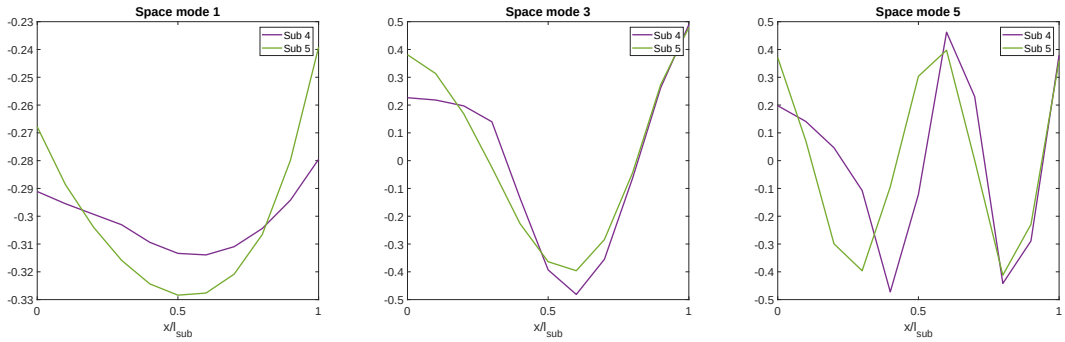


FIGURE 6 – SVD space modes 1, 3 and 5 of frictional forces for substructures 4 and 5.

The introduction of PGD allows the linear stage problems of the LATIN to be solved in a separated representation framework and to save computational time. However, even though finding a new pair of PGD modes is less onerous than solving the problem in full format, adding a new mode to each iteration of the LATIN is still expensive and leads to a high ROB size that can cause the method to lose effectiveness. For this reason, an established way to avoid adding modes at each iteration unless necessary and to simultaneously improve the quality of the reduced representation is to perform a *preliminary stage* by updating all the time functions [9]. However, for strongly non-regular and non-linear problems such as frictional contact problems, this does not prevent the generation of strongly redundant modes that move the PGD basis away from its optimality. It is therefore necessary to contain the size of the PGD basis and control its quality along the iterations of the LATIN. A soft sorting and downsizing algorithm such as the one proposed in [4] may be sufficient, as shown in Figs.7 and 8, where the the first 3 SVD modes of the displacements of substructure 5 are compared with the first 3 PGD modes displaying a good correlation.

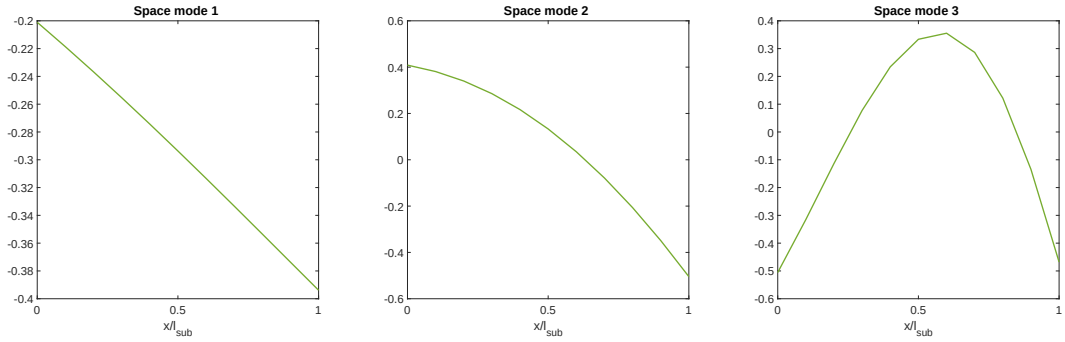


FIGURE 7 – SVD space modes 1, 2 and 3 of displacements for substructure 5.

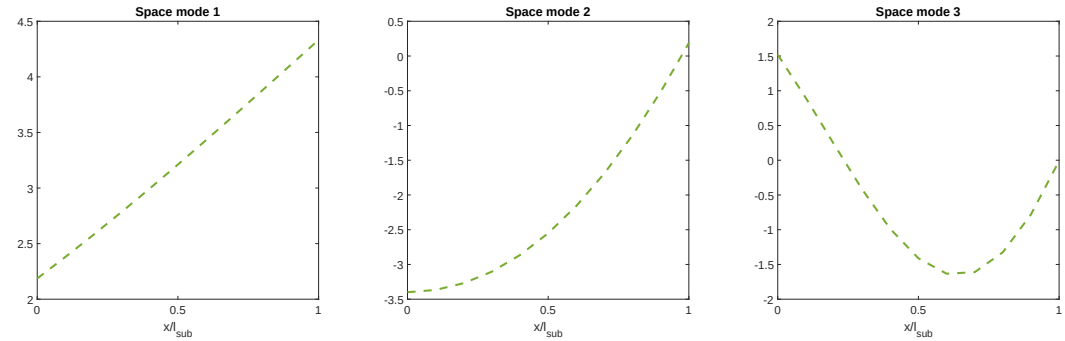


FIGURE 8 – PGD space modes 1, 2 and 3 of displacements for substructure 5.

The non-linear aspect in the LATIN is still integrated in the local stage in full format on the interfaces. This may in fact represent a bottleneck to model reduction, especially in the case of many and large interfaces. One can think of the possibility to integrate the local stage in a reduced format as well by making use, for example, of an a hyperreduction format dedicated to the LATIN-PGD [2].

4 Conclusions

In this paper, some strengths and points of improvement of the LATIN-based multiscale mixed DDM method for the model-order reduction of frictional contact problems were presented. The DDM approach allows to create reduced bases per substructure to better track sliding fronts and propagative phenomena. The multiscale aspect leads to a significant gain in convergence in the early stages, taking advantage of the multiscale nature of the phenomenon itself. However, after few iterations the convergence rate drops and it is necessary to refer to search direction updating techniques, which are scheduled to be proposed. PGD is naturally introduced in the linear stage of LATIN and allows for the addition of new modes where and when required to better follow the evolving frictional phenomena, with the size of the basis which can be efficiently controlled with a downsizing algorithm. The local stage still remains a bottleneck as it is integrated in full format and the potentiality offered by a reduced RPM integration will be considered. Moreover, the need to control the local error with an appropriate convergence indicator is crucial for specific applications. Two-dimensional problems with more complex loadcases and larger number of frictional interfaces will be tested to further comprehend the potentiality of the proposed approach.

Références

[1] F. Bussolati, P.-A. Guidault, M. L. E. Guiton, O. Allix, and P. Wriggers. Robust contact and friction model for the fatigue estimate of a wire rope in the mooring line of a floating offshore wind turbine. *Lecture Notes in Application and Computational Mechanics*, 93 :249–270, 2020.

[2] M. Capaldo, P.-A. Guidault, D. Néron, and P. Ladevèze. The reference point method, a "hyperreduction" technique : Application to PGD-based nonlinear model reduction. *Computer Methods in Applied Mechanics and Engineering*, 322 :483–514, 2017.

[3] Z. Dostál, T. Kozubek, A. Markopoulos, T. Brzobohatỳ, V. Vondrák, and P. Horyl. A theoretically supported scalable TFETI algorithm for the solution of multibody 3D contact problems with friction. *Computer Methods in Applied Mechanics and Engineering*, 205 :110–120, 2012.

[4] A. Giacoma, D. Dureisseix, A. Gravouil, and M. Rochette. Toward an optimal a priori reduced basis strategy for frictional contact problems with LATIN solver. *Computer Methods in Applied Mechanics and Engineering*, 283 :1357–1381, 2015.

[5] P.-A. Guidault, D. Zeka, D. Néron, M. Guiton, and G. Enchéry. Model order reduction for the fatigue life prediction of wire ropes in tension and bending. In *VII International Conference on Computational Contact Mechanics*, 2023.

[6] P. Ladeveze and A. Nouy. On a multiscale computational strategy with time and space homogenization for structural mechanics. *Computer Methods in Applied Mechanics and Engineering*, 192(28-30) :3061–3087, 2003.

[7] P. Ladevèze, A. Nouy, and O. Loiseau. A multiscale computational approach for contact problems. *Computer Methods in Applied Mechanics and Engineering*, 191(43) :4869–4891, 2002.

[8] P. Ladevèze. *Nonlinear Computational Structural Mechanics - new approaches and non-incremental methods of calculation*. Mechanical Engineering Series. Springer New York, 1999.

[9] A. Nouy. A priori model reduction through proper generalized decomposition for solving time-dependent partial differential equations. *Computer Methods in Applied Mechanics and Engineering*, 199(23-24) :1603–1626, 2010.

[10] A. Nouy and P. Ladevèze. Multiscale computational strategy with time and space homogenization : a radial-type approximation technique for solving microproblems. *International Journal for Multiscale Computational Engineering*, 2(4), 2004.

[11] J.-C. Passieux. *Approximation radiale et méthode LATIN multiéchelle en temps et espace*. PhD thesis, Ecole normale supérieure de Cachan, 2008.

[12] R. Ribeaucourt, M.-C. Baietto-Dubourg, and A. Gravouil. A new fatigue frictional contact crack propagation model with the coupled X-FEM/LATIN method. *Computer Methods in Applied Mechanics and Engineering*, 196(33-34) :3230–3247, 2007.

[13] J. C. Simo and T. Laursen. An augmented lagrangian treatment of contact problems involving friction. *Computers and Structures*, 42(1) :97–116, 1992.

[14] D. Zeka, P.-A. Guidault, D. Néron, M. Guiton, and G. Enchéry. Preliminary study for the simulation of wire ropes using a model reduction approach suitable for multiple contacts. In *25ème Congrès Français de Mécanique*, 2022.